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ABSTRACT

This paper presents a new mathematical approach, fuzzy mathematics, for defining, classifying, and analyzing concepts within communications. Fuzzy mathematics is based on the premise that real world phenomena contain characteristics that overlap any artifically created category system or classification rules. Therefore, set membership is characterized by values of a function ranging between zero and one; each value indicates the degree of an element's membership in a given set. Applications of fuzzy mathematics are described for taxonomies in communication, consensus measurement, interaction analysis, and influence hierarchies. (Author/AA)

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NEW METHODOLOGIES FOR COMMUNICATION RESEARCH: APPLICATION OF FUZZY MATHEMATICS*

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The purpose of this paper is to present a new mathematical structure for defining, classifying and analyzing concepts within communication. The assumptions, definitions and applications of fuzzy mathematics have been described (Zadeh, 1965). Fuzzy mathematics is based on the premise that real world phenomena contain characteristics which overlap any artifically created category system or classification rules. Therefore, set membership is characterized by values of a function ranging between zero and one; each value indicating the degree of membership of an element in a given set. Applications of fuzzy mathematics will be described for taxonomies in communication, consensus measurement, interaction analysis and influence hierarchies.

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NEW METHODOLOGIES FOR COMMUNICATION RESEARCH:
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A basic objective of all science is the recognition, definition and classification of variables and concepts. Using classical mathematics, this process usually entails the delineation of fixed and precise variable and class attributes. Concepts, the primary link between theoretical propositions and the empirical world, must be defined so that each class is fully described and completely distinguished from every other class. While this requirement for precise definition and categorization is justified in certain natural sciences, it imposes too stringent a burden on the social sciences and may not allow for an adequate description of the real world (Rapoport, 1958). Hence, it may introduce an element of unrealism or artificiality in social scientific methodology.

At least four important conditions explain why an exact translation of concepts and taxonomies from theory to the empirical world cannot be made. First, social scientists cannot measure their concepts with the same exactness found in the natural sciences. Second, variables are so interdependent that isolation of one or two of the most important of the variables classes imposes an oversimplification upon the process of description, experimentation and understanding. Third, any concept or variable shapes itself differently in each empirical situation where it is found. Fourth, most social scientific concepts possess overlapping attributes making the development of taxonomies with clear-cut categories and category membership rules unrealistic.

Examples of definition and classification problems are replete within the social sciences and the field of communication. Most investigators using interaction analysis, for example, have found that while a given communicative message may reflect one category more strongly than another, it also may reflect at least one of the other categories. Yet the requirements of most interaction analysis systems demand that such categories be considered mutually exclusive and exhaustive. Thus, each utterance may be coded into one and only one category of a dimension. Within the attitude change area, most researchers, while recognizing the imprecise nature of measurement, classify whole groups of their audience into two or possibly three attitude classes based on the direction, salience or intensity of their attitudes. While these researchers acknowledge the importance of considering such constructs as attitude to be continuous, their measurement, analytical and testing procedures usually lead to the classification of audience members reflecting wide ranges of differences into the same overall attitude class: positive, neutral or negative.

A new mathematical structure has been developed to define and manipulate such imprecise concepts as are found in the real world without losing the rigor necessary to quantify any science. This structure, known as fuzzy mathematics, provides a philosophical and logical basis for developing new empirical methodologies for the study of social science phenomena. The purpose of this paper is to introduce the basic structure of fuzzy mathematics and to outline its usefulness to the study, measurement, and classification of various social science concepts.

Introduction to Fuzzy Mathematics

The idea underlying fuzzy mathematics is quite natural. Suppose A represents the set of students at a university who are exactly 6' 0" tall. One's membership in A is completely specified by the characteristic function

$$(1) \qquad f_A(x) = \begin{cases} 1 & 5'11\tfrac{1}{2}" \leq h(x) \leq 6'0\tfrac{1}{2}" \\ 0 & \text{otherwise} \end{cases}$$

where x is any student; h(x) is the height of x; and measurement to $\pm \frac{1}{2}$" is termed "exact". Even though membership in A is imprecise due to our inability to measure exactly, measurement errors of this kind can be handled quite adequately with conventional statistical means, so we call A a "hard" or non-fuzzy set. Quite simply put, x either belongs to A($f_A(s) = 1$), or does not ($f_A(x) = 0$).

Consider now the set A' of all students who are nearly 6'0" tall. Aside from the statistical error incurred when measuring h(x), there is a non-statistical uncertainty about membership of x in A' caused by the fuzzy concept "nearly". This mathematical imprecision can be captured quite readily by extending the range of the membership function f_A of A' from the 2 points 0 and 1 to the entire interval from 0 to 1. Thus we might have

$$f_{A'}(x) = \begin{cases} 1 & 5'11\tfrac{1}{2}" \leq h(x) \leq 6'0\tfrac{1}{2}" \\ .95 & 5'10" \leq h(x)\ 5'11\tfrac{1}{2}" \text{ or } 6'0\tfrac{1}{2}" \leq h(x) \leq 6'2" \\ . & \\ . & \\ . & \end{cases}$$

In other words, $f_{A'}(x) \to 1$ as $h(x) \to 6'\ 0"$. We call the number $f_{A'}(x)$ the grade of membership of x in fuzzy set A'. Clearly if x belongs to A,

then x belongs to A' - but not conversely! The advantage of A' is obvious: we realize a significant gain in information conveyed by our model about the process it represents. Thus from $f_A(y) = 0$ we may infer only that h(y) is not within $\frac{1}{2}$" of 6'0"; but from $f_{A'}(y) = .45$ we know that y is either very short or very tall relative to 6'0". A significant advantage of fuzzy models is that they usually increase both accuracy and usefulness in their representation of real processes.

From this surprisingly simple beginning, several applications of fuzzy mathematical structures have been devised for investigating concepts and relationships in the social sciences. The next section of this paper will illustrate the use of this mathematical structure for investigating consensus in small group decision making, the development of interaction analysis systems, the definition and development of taxonomies relevant to social scientific disciplines, and applications to organizational influence.

Applications of Fuzzy Mathematics

Taxonomies of Communication

The importance and intention behind the development of a taxonomy in any discipline is to provide an accepted general framework to guide research. The taxonomy, complete with rules, provides a justification for dividing a problem area into different levels for analysis and conceptualization, and identifies what factors are included and omitted from a particular area of study. Without a taxonomy, Indik explains that there is

> "...no long run basis for organizing information available from past research, no basis for specifying which variables should be relevant for study when a given problem is proposed. We have no basis for decising which of several overlapping terms should be used as definitions of the variables to be considered, or which of a myriad of operational definitions should be used (1964, 3)."

Classical taxonomies require mutually exclusive categories and demand a rigid set of rules and criteria for classification. Fuzzy taxonomic development, on the other hand, does not require mutual exclusivity. Instead, the variables or concepts of the classification possess a degree of membership in several categories. As a result, a greater degree of information, which could not be considered using a classical approach, is incorporated within the fuzzy classification method. A variable or concept is placed in several categories in a taxonomy and assigned a function to express its degree of membership in the categories. For example, a concept A may be assigned to categories one and two with a membership of .5 in category one and .8 in category two.

One important area of taxonomic development has been the classification of levels of communication. Most researchers assume that communication occurs at five levels: the interpersonal, small group, organizational, public, and mass communication levels. Finding the exact set of definitional and classificatory factors which distinguish these levels, however, has proved difficult. Most researchers agree that a taxonomy based on numbers alone is inadequate. Some texts, for example, define a small group as a communicative situation including from 3-20 persons. Presumably, then, a situation with less than three people is interpersonal and one with more than twenty is public speaking. Other researchers have distinguished among these levels by the amount of intimacy reflected in the exchanged messages. There is no inherent reason, however, why a particularly gutsy and out-going individual cannot self-disclose as intimately over the air waves as others might do in their homes. Feedback has served as a

distinguishing variable for others. Although most descriptions of mass communication admit that feedback is most often extended over a longer period of time for the mass media than for interpersonal communication, it is admitted that this, too, is only a generalization and not a law. In other words, if numbers of people, the existence of intimacy, and the operation of feedback are common to varying degrees to all levels of communication, it is more important to note the degree to which these variables and processes are present at each level rather than trying to use these variables in an all-or-nothing sense to distinguish between these levels.

Similar problems confront the researcher attempting to classify types of small groups and communicator styles. Werbel, Ellis and Fisher (1974) have noted the problems with developing a small group taxonomy based on functions of the groups. While families may serve largely an educational function, they also serve social, task, therapeutic and other functions. While decision making groups may exist largely for the accomplishment of a given task, they are simultaneously fulfilling social functions. Warbel, et al., have described a taxonomy based on the amount of information flowing between the group and the subsystems above and below it in a larger organizational system. Here again, the important point is not that some groups always communicate freely with their environment while others are virtually walled off from it, but the degree to which this free flow of communication exists. A fuzzy taxonomy would allow for the definition and distinction among group types according to the degree to which groups exhibit all meaningful characteristics outlined in taxonomic criteria.

Fuzzy Measures of Consensus

Most consensus measures have been based on the members' agreement with the final decision (Knutson, 1972). While the final agreement on a chosen alternative is important, this measure minimizes information concerning the strength of that agreement as well as member feelings about all other alternatives discussed during decision making. To incorporate this wide range of information, Spillman, Spillman and Bezdek (1977) and Bezdek, Spillman and Spillman (1977) have used fuzzy mathematics to measure consensus in small decision making groups.

Rather than forcing group members to totally accept one alternative and totally reject all others at the end of a group discussion, the fuzzy measure recognizes that members may have degrees of acceptance and rejection of all alternatives. The fuzzy consensus measure requires that the list of decision alternatives be placed in an n by n matrix. Members are asked to rate their personal preference for the topic labeled in the column over that labeled in the row for each square of the bottom triangle of the matrix, excluding the diagonal. This scale indicates the individual participant's preference for every one of their alternatives over all the others. Members are told to fill in any number between zero and one in the matrix squares. A number between .50 and 1.00 indicates a relative preference for the column alternative over the row alternative. A .50 indicates no preference between alternatives, and a number between .49 and 0.00 indicates a relative preference for the row alternative over the column alternative.

An average fuzzy relation matrix can be constructed by averaging the entries in each square for all members of the group. Using the average

matrix, a measure of the degree of fuzziness in the overall group's decision can be calculated using formula (1):

$$F(R) = \frac{2\ tr(R^2)}{n(n-1)} \quad (1)$$

where tr(B) is the trace of matrix B (the sum of the diagonal elements). The fuzzy measure ranges from zero to $\frac{1}{2}$ where zero indicates complete consensus among group members; i.e., no fuzziness or disagreement with the final decision proposal or the rating of all other decision alternatives and $\frac{1}{2}$ indicates a state of total indecision, i.e., every alternative is equally palatable. An example of a ten by ten alternative preference matrix is illustrated in figure one.

The fuzzy consensus measure has at least three advantages over most existing measures. First, it appears to provide a more realistic way of observing and measuring an individual's thought process during the group decision making process. Second, it provides a means of analyzing the overall structure of the entire decision making process rather than narrowing the focus to the one chosen solution. Finally, it is based on a ratio scale which allows for statistical and mathematical manipulations not possible with typical ordinal or interval scaling techniques.

Interaction Analysis

A major goal of interaction analysis is to systematically analyze the forms, functions and patterns of communication by classifying each communicative message into a descriptive set of categories. Most researchers simplify the coding scheme by forcing each utterance or message to be classified into one category of each relevant dimension. Thus, each message is either a member or not a member of the set defined by the category definition.

Several researchers have noted that communicative messages are more a combination of form, content and function than a reflection of one and only one category. Spillman (1976) criticized the Ellis (1976) coding system on the grounds that many comments served at least two relational functions. She found that coding problems could not be satisfied by dividing a communicative utterance into parts, for comments would often serve different purposes within the same sentences or short phrases. Knapp, Hart, Friedrich and Shulman (1973) attempted to avoid this categorization problem by allowing coders to indicate as many categories in a given dimension as they felt were contained within a given communicative utterance. Although this procedure surmounts the problem of single-category coding and provides more information concerning the total nature of a comment, it is not wholly satisfactory. Each comment is still coded with a zero or a one indicating that the given characteristic or function is totally present or totally absent. No indication of the proportion of the utterance devoted fifferentially to all categories is noted.

A fuzzy interaction analysis system would allow a coder to assign a membership value between zero and one to a message for one or more categories in a given dimension. If a verbal utterance contains an assertion, but ends with a question, it could be coded as a .75 assertion and .4 question. In general the sum of the membership values across categories does not have to total 1.00 because this is not a probability measure. The characteristics that place a message in a category are independent of the characteristics that related to other categories.

Analysis of a fuzzy interaction analysis system could be accomplished with similar methods now being used for standard systems. Fuzzy Markov

chains are currently under investigation and should be developed for statistical use soon.

Fuzzy interaction analysis systems have the advantages of flexibility, high information value, and realism. Such a system could overcome many of the constraining conceptual and operational problems encountered by present procedures.

Influence in Organizations

Thomason and Marinos (1973) have applied fuzzy mathematics to the study of informal and formal influence in organizations. While formal relationships are nonfuzzy, an individual is by definition a superior or subordinate relative to another individual, informal relations are specified by such fuzzy concepts as "likes","advises" and "communicates with." These informal relations result in lines of influence characterized by a fuzzy influence matrix as shown below for a four-person organization.

$$R_o = \begin{bmatrix} 1 & .1 & .6 & .5 \\ .2 & .4 & .2 & .5 \\ .3 & .3 & 1 & .2 \end{bmatrix}$$

$R_o(p_1, p_2) = .1$ measures the degree of informal influence of person one over person two. The formal superior-subordinate relationship is given by a nonfuzzy matrix as shown below.

$$R_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$R_1(p_1, p_2) = 1$ means that person two is directly subordinate to person one.

Thomason and Marinos have used these matrices theoretically and conceptually to describe the relationship between formal and informal influence hierarchies within organizations. Actual empirical research has yet to be reported. This research could be helpful beyond organizational research, for the study of social influence permeates the entire realm of the social sciences. Influence is rarely an all-or-nothing process. A fuzzy mathematical function best captures the degree of influence which actually exists between people.

Conclusion

Recognizing the importance of taxonomies, definitions and concepts within the social sciences, the purpose of this paper was to provide a methodological approach which would overcome many problems inherent to the normal classification process adopted from classical mathematics and the physical sciences. Realizing that social scientific phenomena reflect many combinations and degrees of attributes and characteristics, we have presented a fuzzy mathematical structure which allows a researcher to include complex and overlapping information rather than eliminating it for simplicity's sake. Fuzzy mathematics is based upon the premise that real world phenomena contain characteristics which overlap any artificially created category systems and classification rules. Therefore, in fuzzy mathematics an element of a set is characterized by a function between zero and one indicating its degree of membership in a given set.

Applications of this remarkable simple principle have been illustrated for the development of communicative taxonomies, measurement of consensus, development of interaction analysis systems, and the analysis of influence

hierarchies in organizations. While empirical work has been pursued only in the area of consensus development in small group decision making, this methodological approach should prove extremely useful to many researchers in all areas.

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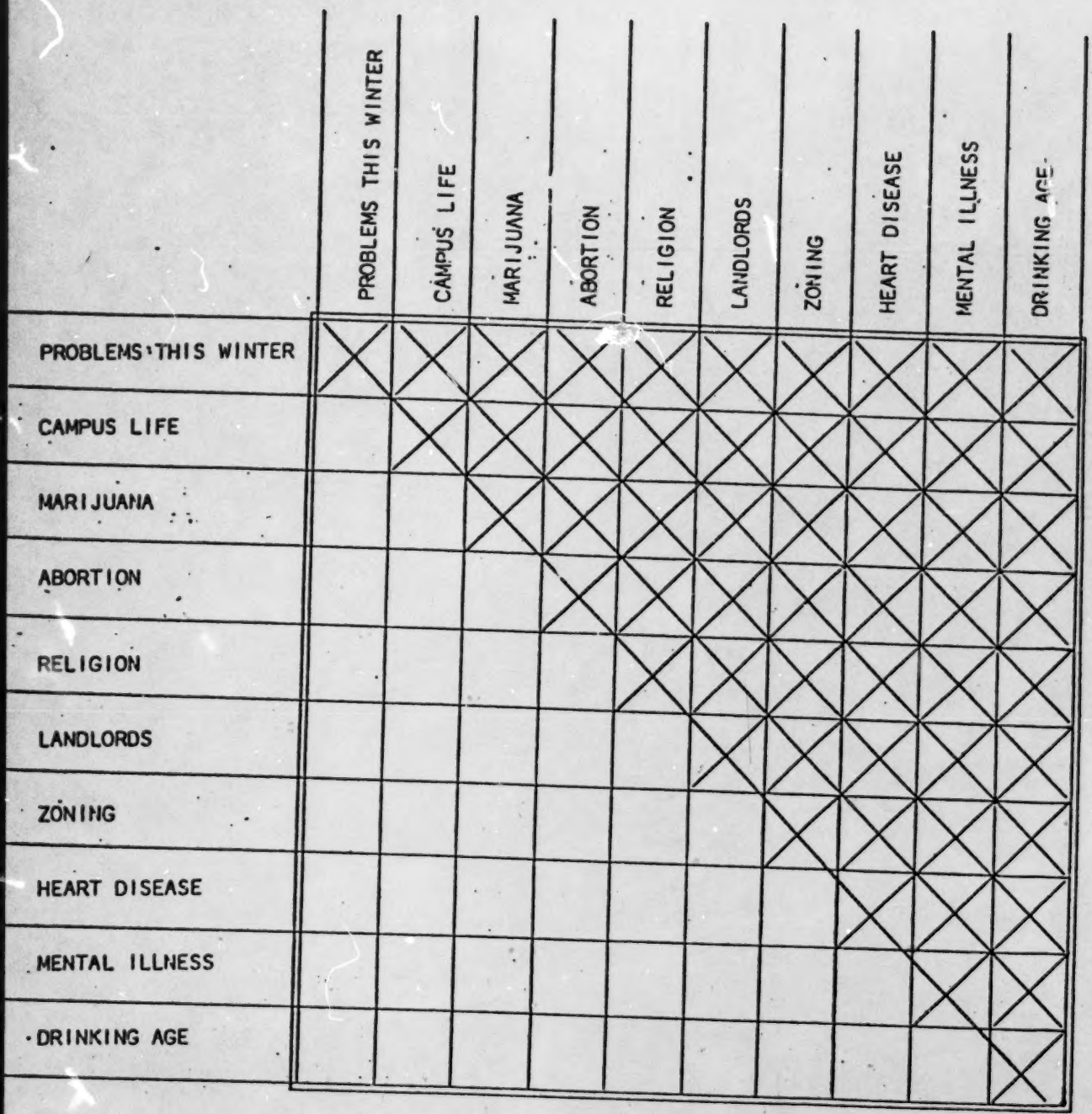

Figure 1: Measuring Instrument for Group One